class- X, Sub: Maths, chap-1 (Real numbers) 26.3.20

Introduction:-

Lemma: Lemma is a proven statement used for proving another statement.

Algorithm:- An algorithm is a series of well defined steps which gives a procedure for solving a type of problem.

Euclid's Division Lemma: Given positive integers a and b there exist unique integers q and r satisfying $a = bq + r$, $0 \le r < b$

Q. Use Euclid's division algorith, to find HCF of 960 and 432

on applying Euclid's division lemma for 960 and 432

we get $960 = 432 \times 2 + 96$

Here remainder $= 96 \neq 0$

$432 = 96 \times 4 + 48$, remainder $= 48 \neq 0$

$96 = 48 \times 2 + 0$

Hence remainder $= 0$, and last divisor $= 48$

$\therefore$ HCF of 960 and 432 is 48

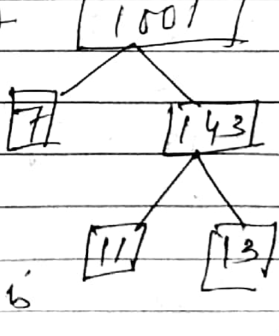
Fundamental Theorem of Arithmetic:-

According to the fundamental th. of arithmetic, every composite number can be written as the product of primes and this factorisation is unique.

Q. Express 1001 as a product of its prime factors

Factor Tree method

~~1000~~



$\therefore 1001 = 7 \times 11 \times 13$

Q. Explain why $3 \times 5 \times 7 + 7$ is a composite number?

$3 \times 5 \times 7 + 7 = 105 + 7 = 112$

prime factor of $112 = 2 \times 2 \times 2 \times 2 \times 7 = 2^4 \times 7$

So, it is the product of prime factors 2 and 7

i.e. it has factors other than 1 and the number itself

Hence it is a composite number.

Relation between numbers and their HCF and LCM.

For any two positive integers a and b,

$$HCF(a,b) \times LCM(a,b) = a \times b$$

$$HCF(a,b) = \frac{a \times b}{LCM(a,b)}$$

Ex. If HCF of two numbers is 2 and their product is 120, find their LCM

Ans. $LCM(a,b) = \frac{\text{Product of 2 number}}{HCF(a,b)}$

$$= \frac{120}{2} = 60$$

Ex. $LCM(a,b,c) = \frac{a \times b \times c \times HCF(a,b,c)}{HCF(a,b) \times HCF(b,c) \times HCF(c,a)}$

Defn: Rational numbers

A number that can be expressed as $\frac{p}{q}$, where p, q are integers and $q \neq 0$ is called rational numbers. eg. $\frac{3}{5}$, $\frac{-7}{3}$ etc.

Irrational Numbers: A number that cannot be expressed in the form $\frac{p}{q}$ where p, q are integers and $q \neq 0$ is called an irrational number.

eg. $\sqrt{2}$, $\sqrt{3}$, π etc.

Real number: A number which is either rational or irrational, is called a real number.

Th: Let p be a prime number and a be a positive integer. If p divides a^2, then p divides a.

Q. Show that $3\sqrt{2}$ is an irrational number.

Let us assume that $3\sqrt{2}$ be a rational.

Let $3\sqrt{2} = \frac{p}{q}$ p, q are int. and $q \neq 0$

$\Rightarrow \sqrt{2} = \frac{p}{3q}$, Since p is an integer and 3q is also an integer

$\therefore \frac{p}{3q}$ is a rational number

$\Rightarrow \sqrt{2} =$ rational

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But it contradicts the fact that $\sqrt{2}$ is rational.
Thus our assumption is wrong.
Hence $3\sqrt{2}$ is an irrational number.

Q. Check whether the rational number $\frac{1}{13}$ has terminating recurring or non-terminating recurring decimal expansion.
Here prime factor of 13 are not of the form $2^n.5^m$. So, it will not have a terminating decimal expansion.

Q. Use Euclid's division lemma to show that square of any positive integer is either of the form $3m$ or $3m+1$, for some integer m.
Ans By Euclid's division lemma we know
$a = bq + r$ $\quad 0 \le r < b$ [a, b are positive integer]
Let $b = 3$ then a will be of the form
$3q, 3q+1, 3q+2$ [when we divide a number by 3 remainder may be 0, 1, 2]
Let $a = 3q$ $\therefore a^2 = 9q^2 = 3.3q^2 = 3m$ ...(I) where $m = 3q^2$

Also $a = 3q+1$
$\therefore a^2 = (3q+1)^2 = 9q^2 + 1 + 6q$
$= 3q(3q+2) + 1$
$= 3m + 1$ ...(II) where $m = 3q(3q+2)$

And $a = 3q+2$
$\therefore a^2 = (3q+2)^2 = 9q^2 + 4 + 12q = 9q^2 + 12q + 3 + 1$
$= 3(3q^2 + 4q + 1) + 1 = 3m + 1$ ...(III)
where $m = 3q^2 + 4q + 1$

From (I), (II), (III)
We get $a^2 = 3m$ or $3m+1$

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Sub - Maths Assessment - 1 (Real numbers)

Q.1. what is the HCF of smallest prime numbers and the smallest composite number ?

Q.2. A number when divided by 61 gives 27 as the quotient and 32 as remainder. Find the number.

Q.3. Find the largest number which divides 70 and 125 leaving remainder 5 and 8, respectively.

Q.4. Can two numbers have 18 as their HCF and 380 as their LCM ? Give reason

Q.5. Given that $\sqrt{2}$ is irrational, prove that $5+3\sqrt{2}$ is an irrational number.

Q.6. Check whether 15^n can end with digit zero for any natural number n.

Q.7. A rational number in its decimal expansion is 327.7081. What can you say about the prime factor of q. Give reason.

Q.8. Find the least number that is divisible by all the numbers from 1 to 5 (both inclusive)

Q.9. Find HCF and LCM of 404 and 96 and verify that HCF X LCM = product of the two given numbers

Q.10 Prove that one of any three consecutive positive integers must be divisible by 3.

Q.11 Use Euclid's division algorithm, to find the largest number, which divides 957 and 1280 leaving remainder 5 in each case.

Q.12. If n is an odd integer, then show that n^2-1 is divisible by 8.

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