Self-Evaluation Test

Time Allowed : 1 hour 30 minutes] [*Maximum Marks* : 55

1. Show that the relation R in the set $\{1, 2, 3\}$, given by $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)\}$ is reflexive but not symmetric. 1
2. Show that the binary operation $* : R \to R$ given by $a * b = a + 2b$ is not commutative. 1
3. Let $f : N \to N$ defined by $f(x) = 3x$. Show that 'f' is not an onto function. 1
4. Let n be a fixed positive integer. Define a relation R in Z as follows : $\forall\ a, b \in Z$, $aRb \Leftrightarrow a - b$ is divisible by n. Show that R is an equivalence relation. 4
5. Show that $f : [-1, 1] \to R$, given by $f(x) = \dfrac{x}{x+2}$, $x \neq -2$ is one-one. Find the inverse of function $f : [-1, 1] \to R_f$. 4
6. Show that the number of equivalence relations in the set $\{1, 2, 3\}$ containing $(1, 2)$ and $(2, 1)$ is two. 4
7. A relation $R : N \to N$ is given by $R = \{(a, b) : b$ is divisible by $a\}$. Check whether R is an equivalence relation. 4
8. Show that the relation $R : N \to N$ defined by $(a, b)\ R\ (c, d) \Leftrightarrow a + d = b + c$ for all $(a, b), (c, d) \in N \times N$ is an equivalence relation. 4
9. Let R be the set of real numbers and $*$ be the binary operation defined on R as $a * b = a + b - ab$, $\forall\ a, b \in R$. Find the identity element with respect to binary operation $*$. 4
10. Let $*$ be a binary operation on N, given by $a * b =$ l.c.m. (a, b) for $a, b \in N$. Find : (*i*) $2 * 4$, (*ii*) $3 * 5$, (*iii*) Is '$*$' associative ? 4
11. If $A = \{a, b, c, d\}$ and $f = \{(a, b), (b, d), (c, a), (d, c)\}$, show that f is one-one from A onto A. Find f^{-1}. 4
12. Show that the function $f : R \to R$ defined by $f(x) = \dfrac{x}{x^2+1}$, $\forall\ x \in R$ is neither one-one nor onto. 4
13. Let relation R, on the set of natural numbers N is defined as follows : $R = \{(x, y) \in N \times N : 2x + y = 41\}$. Find the domain and range of the relation R. Also verify whether R is reflexive, symmetric and transitive. 4
14. Show that $f : N \cup \{0\} \to N \cup \{0\}$ given by $f(n) = \begin{cases} n+1, & \text{if } n \text{ is even} \\ n-1, & \text{if } n \text{ is odd} \end{cases}$ is a bijective function. 6
15. Let $A = N \times N$ and let $*$ be a binary operation on A defined by $(a, b) * (c, d) = (ac, bd)$. Show that (*i*) $(A, *)$ is commutative (*ii*) $(A, *)$ is associative. Find the identity element, if any, in A. 6

ANSWERS

5. $f^{-1}(x) = \dfrac{2x}{1-x}$
7. not an equivalence relation
9. 0
10. (*i*) 4 (*ii*) 15 (*iii*) yes, associative
11. $f^{-1} = \{(b, a), (d, b), (a, c), (c, d)\}$
13. Domain $= \{1, 2, 3, 4, ..., 20\}$, Range $= \{1, 3, 5, 7, ..., 39\}$
Neither reflexive, nor symmetric, nor transitive.
15. (1, 1)

Self-Evaluation Test

Time Allowed : 1 hour 30 minutes] [*Maximum Marks* : 40

1. If $\sin\left\{\sin^{-1}\frac{1}{5}+\cos^{-1}x\right\} = 1$, then find the value of x. 1

2. Find the value of, $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$. 1

3. Prove that, $\sin^{-1}x = \cos^{-1}\sqrt{1-x^2}$. 1

4. Evaluate, $\tan^{-1}(-\sqrt{3})$. 1

5. Represent, $\sin^{-1}(2ax\sqrt{1-a^2x^2})$, $-\frac{1}{\sqrt{2}} \le ax \le \frac{1}{\sqrt{2}}$ in the simplest form. 4

6. Write the function, $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$, $x \neq 0$ in the simplest form. 4

7. Find the value of, $\tan\left[\frac{1}{2}\left\{\sin^{-1}\frac{2x}{1+x^2}+\cos^{-1}\left(\frac{1-y^2}{1+y^2}\right)\right\}\right]$, $|x| < 1$, $y > 0$, $xy < 1$. 4

8. Prove that, $\sin^{-1}x + \sin^{-1}y = \sin^{-1}\left[x\sqrt{1-y^2}+y\sqrt{1-x^2}\right]$. 4

9. Find the value of, $\tan^{-1}\left[2\cos\left\{2\sin^{-1}\frac{1}{2}\right\}\right]$. 4

10. Show that $\sin^{-1}\frac{3}{5} - \sin^{-1}\frac{8}{17} = \cos^{-1}\frac{84}{85}$. 4

11. Prove that, $\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right) = \frac{x}{2}$, $x \in \left(0, \frac{\pi}{4}\right)$. 4

12. Write the function, $\cot^{-1}(\sqrt{1+x^2}+x)$ in the simplest form. 4

13. If $\cos^{-1}\frac{x}{2} + \cos^{-1}\frac{y}{3} = \theta$, then prove that $9x^2 - 12xy\cos\theta + 4y^2 = 36\sin^2\theta$. 4

ANSWERS

1. $\frac{1}{5}$ 2. $\frac{5\pi}{6}$ 4. $-\frac{\pi}{3}$ 5. $2\sin^{-1}(ax)$ 6. $\frac{1}{2}\tan^{-1}x$

7. $\frac{x+y}{1-xy}$ 9. $\frac{\pi}{4}$ 12. $\frac{x}{2}$

Time Allowed : 1 hour 30 minutes] [Maximum Marks : 50

1. Find the sum of matrix $A = \begin{pmatrix} 2 & -1 \\ 4 & 6 \end{pmatrix}$ and its additive inverse. 1

2. For the matrix A, show that $A - A^T$ is a skew-symmetric matrix. 1

3. Given a matrix $A = [a_{ij}]$, $1 \le i \le 3$ and $1 \le j \le 3$, where $a_{ij} = i + 2j$. Write the element
   (i) a_{11} (ii) a_{32}
   (iii) a_{23} (iv) a_{34}. 4

4. If $A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $B = [2 \; -3 \; 4]$, find AB. 4

5. If $A = \begin{pmatrix} 3 & -2 \\ 4 & -2 \end{pmatrix}$ and $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, then find k if $A^2 = kA - 2I$. 4

6. Find a matrix X, such that $A + 2B + X = 0$, where $A = \begin{bmatrix} 2 & -1 \\ 3 & 5 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 1 \\ 0 & 2 \end{bmatrix}$. 4

7. Find X and Y, given that $3X - Y = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ and $X - 3Y = \begin{bmatrix} 0 & -1 \\ 1 & -1 \end{bmatrix}$. 4

8. If $A = \begin{bmatrix} 3 & 4 \\ -4 & -3 \end{bmatrix}$, find $f(A)$, where $f(x) = x^2 - 5x + 7$. 4

9. Find the inverse using elementary transformations, if exists, for the matrix $\begin{bmatrix} 8 & -4 \\ -2 & 1 \end{bmatrix}$. 4

10. Find the values of p and q such that $A^2 + pI = qA$, where $A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$. 4

11. Find the value of x, $x \in I$ such that $[x \;\; 4 \;\; -1] \begin{bmatrix} 2 & 1 & -1 \\ 1 & 0 & 0 \\ 2 & 2 & 4 \end{bmatrix} [x \;\; 4 \;\; -1]^T = 0$. 4

12. Prove the following by principle of Mathematical Induction, if $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ then,
    $A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$ for every positive integer n. 6

13. Using elementary transformations, find the inverse of matrix $\begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$. 6

Self-Evaluation Test

Time Allowed : 1 hour 30 minutes] [*Maximum Marks* : 55

1. If $\begin{vmatrix} 2x & 4 \\ -1 & x \end{vmatrix} = \begin{vmatrix} 6 & -3 \\ 2 & 1 \end{vmatrix}$, find x. 1

2. Using properties of determinants, show that $\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix} = 0$. 1

3. Without actual expansion, prove that : $\begin{vmatrix} 0 & 99 & -998 \\ -99 & 0 & 997 \\ 998 & -997 & 0 \end{vmatrix} = 0$. 1

4. Find the matrix A, such that $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. 4

5. Without expanding the determinant, prove that : $\begin{vmatrix} x+y & x & x \\ 5x+4y & 4x & 2x \\ 10x+8y & 8x & 3x \end{vmatrix} = x^3$. 4

6. If $A = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 6 & 7 \\ 8 & 9 \end{bmatrix}$, verify that $(AB)^{-1} = B^{-1}A^{-1}$. 4

7. Show that : $\begin{vmatrix} a^2 & 2ab & b^2 \\ b^2 & a^2 & 2ab \\ 2ab & b^2 & a^2 \end{vmatrix} = (a^3 + b^3)^2$. 4

8. Find A^{-1}, if $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$. Also show that $A^{-1} = \dfrac{A^2 - 3I}{2}$. 4

9. Prove that $\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix} = x^2(x + a + b + c)$. 4

10. Prove that : $\begin{vmatrix} ab & -b^2 & bc \\ ca & bc & -c^2 \\ -a^2 & ab & ca \end{vmatrix} = 4a^2b^2c^2$. 4

11. Prove that : $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ bc & ca & ab \end{vmatrix} = (a-b)(b-c)(c-a)(ab+bc+ca)$. 4

12. Using properties of determinants, solve for x : $\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$. 4

13. Using matrix method, solve the following system of linear equations : 6

$x + y + z = 3;\ 2x - y + z = 2\ ;\ x - 2y + 3z = 2$

14. Show that the following determinant vanishes :

$$\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}.$$ 4

15. Find the product of matrices $A = \begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$ and use it for solving the equations : $x + y + 2z = 1,\ 3x + 2y + z = 7,\ 2x + y + 3z = 2$. 6

ANSWERS

1. $x = \pm 2$

4. $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

8. $A^{-1} = \frac{1}{2}\begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$

12. $x = 0, 3a$

13. $x = 1, y = 1, z = 1$

15. $AB = 4I\ ;\ x = 2,\ y = 1,\ z = -1$

Self-Evaluation Test

Time Allowed : 1 hour 30 minutes] [*Maximum Marks* : 55

1. Examine the continuity of the function $f(x) = \dfrac{1}{x-5}, x \in R$. 1

2. If $y = \sec^{-1}\left(\dfrac{x-1}{x+1}\right) + \sin^{-1}\left(\dfrac{x+1}{x-1}\right)$, find $\dfrac{dy}{dx}$. 1

3. Find y'', if $y = \sin x$. 1

4. Test the continuity of the function

$$f(x) = \begin{cases} (x-a)\sin\left(\dfrac{1}{x-a}\right), & x \neq a \\ 0, & x = a \end{cases} \quad \text{at } x = a.$$

4

5. Find the values of a and b, such that the function defined by $f(x) = \begin{cases} 5, & x \le 2 \\ ax+b, & 2 < x < 10 \\ 21, & x \ge 10 \end{cases}$

is a continuous function. 4

Differentiate w.r.t. x.

6. $\dfrac{e^{2x} + e^{-2x}}{e^{2x} - e^{-2x}}$. 4

7. $y = \sin^{-1}\left(\dfrac{3\sin x + 4\cos x}{5}\right)$. 4

8. $(\sin x)^x + \sin^{-1}\sqrt{x}$. 4

9. Find the derivative of $\sin^{-1}\left(\dfrac{1-x}{1+x}\right)$, w.r.t. $\sqrt{x}$. 4

10. Prove that : $\dfrac{d}{dx}\left[\dfrac{x}{2}\sqrt{a^2-x^2} + \dfrac{a^2}{2}\sin^{-1}\dfrac{x}{a}\right] = \sqrt{a^2-x^2}$. 4

11. Find $\dfrac{d^2y}{dx^2}$, if $x = at^2, y = 2at$. 4

12. Verify LMV theorem for the function : $y = \sqrt{x-2}$ in [2, 6]. 4

13. If $x = a(\theta - \sin\theta), y = a(1 - \cos\theta)$, find $\dfrac{d^2y}{dx^2}$ at $\theta = \dfrac{\pi}{2}$. 4

14. Find $\frac{dy}{dx}$, for the function $x^y + y^x = 1$. 6

15. If $y = (\cot^{-1} x)^2$, show that $(x^2 + 1)^2 y_2 + 2x(x^2 + 1)y_1 - 2 = 0$. 6

ANSWERS

1. Not continuous at $x = 5$
2. 0
3. $-\sin x$
4. Continuous
5. $a = 2, b = 1$
6. $\frac{-8}{(e^{2x} - e^{-2x})^2}$
7. 1
8. $(\sin x)^x [x \cot x + \log(\sin x)] + \frac{1}{2\sqrt{x}\sqrt{1-x}}$
9. $\frac{-2}{1+x}$
11. $\frac{-1}{2at^3}$
12. $c = 3$
13. $-\frac{1}{a}$
14. $-\frac{yx^{y-1} + y^x \log y}{x^y \log x + xy^{x-1}}$

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Ex. 1.

Sol.

Ex. 2.

Sol.

Ex. 3.

Self-Evaluation Test

Time Allowed : 1 hour 30 minutes] [*Maximum Marks* : 55

1. Prove that the function $f(x) = x^3 + x^2 + x + 1$ does not have a maxima or minima. 1
2. The cost function of a firm is given by $C(x) = 0.005x^3 - 0.\ 2x^2 - 30x + 200$, where x is the output. Find the marginal cost. 1
3. The total revenue received from the sale of x units of a product is given by $R(x) = 13x^2 + 26x + 15$. Find the marginal revenue when $x = 7$. 1
4. A particle moves along the curve $y = \frac{2}{3}x^3 + 1$. Find the points on the curve at which the y-coordinate changes twice as fast as x-coordinate. 4
5. Show that the function $f(x) = \tan^{-1}(\sin x + \cos x)$, $x > 0$ is strictly decreasing on the interval $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$. 4
6. Find the approximate value of $\sqrt[5]{31.9}$, using differentials. 4
7. Water is running into a conical tank of height 10 m and diameter 10 m at the top, at a constant rate of 18 m^3/min. How fast is the water rising in the tank at any instant ? 4
8. Find the equation of the normal at the point (am^2, am^3) for the curve $ay^2 = x^3$. 4
9. Two equal sides of an isosceles triangle with fixed base b are decreasing at the rate of 3 cm/sec. How fast is the area decreasing when the two equal sides are equal to the base ? 4
10. Find the interval(s) for which the function $f(x) = \log(2 + x) - \frac{2x}{2 + x}$ is increasing or decreasing. 4
11. A closed circular cylinder has a volume of 2156 cu. cm. What will be the radius of its base so that its total surface area is minimum ? $\left[\text{Take } \pi = \frac{22}{7}\right]$ 4
12. Find the point on the curve $y^2 = 4x$ which is nearest to the point $(2, -8)$. 4
13. Find the approximate value of $y = 2(4.02)^2 - 3(4.02)^{3/2}$. 4
14. Find the maximum volume of a cylinder, generated by rotating a rectangle of perimeter 48 cm about one of its sides. 6
15. A point on the hypotenuse of a right-angled triangle is at distances a and b from the sides of the triangle. Show that the minimum length of the hypotenuse is $(a^{\frac{2}{3}} + b^{\frac{2}{3}})^{\frac{3}{2}}$. 6

ANSWERS

2. $0.015x^2 - 0.4x - 30$
3. 208
4. $\left(-1, \frac{1}{3}\right), \left(1, \frac{5}{3}\right)$
6. 1.999
7. $\frac{72}{\pi h^2}$ units
8. $2x + 3my - am^2(2 + 3m^2) = 0$
9. $\sqrt{3}b$ cm^2/sec
10. Increasing $(2, \infty)$, decreasing $(-2, 2)$
11. 7 cm.
12. $(4, -4)$
13. 8.14
14. 2048π cm^3

Self-Evaluation Test

Time Allowed : 1 hour 30 minutes] [*Maximum Marks* : 55

Evaluate each of the following integrals :

1. $\int \log x \, dx$. 1
2. $\int_0^1 e^{2-5x} \, dx$. 1
3. $\int_{-1}^{1} \log\left(\frac{4-x}{4+x}\right) dx$. 1
4. $\int \sin 2x \sin 5x \, dx$. 1
5. How will you proceed to evaluate, $\int_{-1}^{1} |x| \, dx$? 1
6. $\int \frac{(1+\sin x)\, e^x}{(1+\cos x)} \, dx$. 4
7. $\int \frac{\sin x}{\sin(x+\alpha)} \, dx$. 4
8. $\int \left\{\log(\log x) + \frac{1}{(\log x)^2}\right\} dx$. 4
9. $\int \frac{dx}{x[6(\log x)^2 + 7\log x + 2]}$. 4
10. $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\cot^{3/2} x}$. 4
11. $\int_1^4 (|x-1| + |x-2| + |x-3|) \, dx$. 6
12. $\int_{-1}^{3/2} |x \sin \pi x| \, dx$. 6
13. $\int_1^2 (2x^2 + x + 7) \, dx$, as a limit of sums. 6
14. Prove that $\int_0^a \sin^{-1}\sqrt{\frac{x}{a+x}} \, dx = \frac{a}{2}(\pi - 2)$. 6
15. Prove that $\int_{-a}^{a} \sqrt{\frac{a-x}{a+x}} = a\pi$. 6

ANSWERS

1. $x \log x - x + c$
2. $\frac{1}{5}\left[e^2 - e^{-3}\right]$
3. 0
4. $-\frac{1}{2}\left[\frac{\sin 7x}{7} - \frac{\sin 3x}{3}\right] + c$
5. $\int_{-1}^{0} (-x)\, dx + \int_0^1 x \, dx$ and integrate
6. $e^x \tan \frac{x}{2} + c$
7. $x \cdot \cos\alpha - \sin\alpha \cdot \log|\sin(x+\alpha)| + c$
8. $x\left[\log(\log x) - \frac{1}{\log x}\right] + c$
9. $\log\left|\frac{1+2\log x}{2+3\log x}\right| + c$